

SOLUTION

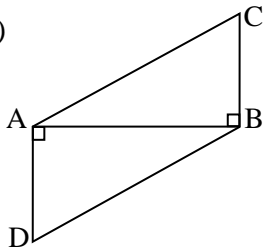
Q.1. (A) Four alternative answers are given for every sub-question. Select the correct alternative and write the alphabet of that answer. [4]

- (1) From the following points point lies to the right side of the origin on X-axis.
(a) $(-2, 0)$ (b) $(0, 2)$ (c) $(2, 3)$ (d) $(2, 0)$ [1]
- (2) $\Delta PQR \sim \Delta STU$ and $A(\Delta PQR) : A(\Delta STU) = 64:81$, then what is the ratio of corresponding sides?
(a) 8:9 (b) 64:81 (c) 9:8 (d) 16:27 [1]
- (3) In a right-angled triangle, if the sum of the squares of the sides making right angle is 169, then what is the length of hypotenuse?
(a) 15 (b) 13 (c) 5 (d) 12 [1]
- (4) If $\tan \theta = \sqrt{3}$, then the value of θ is
(a) 60° (b) 30° (c) 90° (d) 45° [1]

Ans. (1) - (d), (2) - (a), (3) - (b), (4) - (a). [4]

Q.1. (B) Solve the following sub-questions.**[4]**

(1)



In the above figure, seg $CB \perp$ seg AB ,
 seg $AD \perp$ seg AB . If $BC = 4$, $AD = 8$,
 then find $\frac{A(\Delta ABC)}{A(\Delta ADB)}$.

Solution:

$$\frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{\frac{1}{2} \times AB \times BC}{\frac{1}{2} \times AB \times AD}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{BC}{AD} \quad \dots[1/2]$$

$$\left. \begin{array}{l} BC = 4 \\ AD = 8 \end{array} \right\} \dots\dots\dots \text{(given)}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{4}{8}$$

Ans.

$$\boxed{\frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{1}{2}}$$

...[1/2] [1]

(2) Find the co-ordinates of the mid-point of the segment joining the points (22, 20) and (0, 16).

Solution:

Suppose $(x_1, y_1) \equiv (22, 20)$ and $(x_2, y_2) \equiv (0, 16)$ and the co-ordinates of the mid-point are (x, y) .

\therefore By the mid-point formula,

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \dots[1/2] \\ &= \left(\frac{22 + 0}{2}, \frac{20 + 16}{2} \right) \end{aligned}$$

$$\therefore \boxed{(x, y) = (11, 18)} \quad \dots[1/2] [1]$$

Ans. The co-ordinates of the mid-point are (11, 18).

- (3) Two circles having radii 7 cm and 4 cm touch each other internally. Find the distance between their centres.

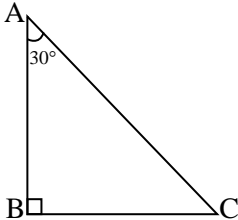
Solution:

The two circles touch each other internally.

$$\begin{aligned} \therefore \text{The distance between their centres} &= 7 \text{ cm} - 4 \text{ cm} \\ &= 3 \text{ cm} \quad \dots[1/2] \\ &\dots\dots \text{(By theorem of touching circles)} \end{aligned}$$

Ans. The distance between the centres of the given circle is 3 cm.
...[1/2] [1]

(4)



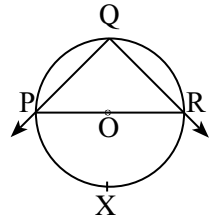
In ΔABC , $\angle B = 90^\circ$, $\angle A = 30^\circ$,
 $AC = 14$, then find BC .

Solution:

$$\begin{aligned} \angle B &= 90^\circ, \angle A = 30^\circ \dots\dots\dots \text{(given)} \\ \therefore \angle C &= 60^\circ \dots\dots\dots \text{(Remaining angle of } \Delta ABC) \\ \therefore \Delta ABC &\text{ is a } 30^\circ - 60^\circ - 90^\circ \text{ triangle.} \\ \therefore \text{By the } 30^\circ - 60^\circ - 90^\circ &\text{ triangle theorem,} \\ \text{Side opposite the } 30^\circ &\text{ angle} = \frac{1}{2} \times \text{hypotenuse} \\ \therefore BC &= \frac{1}{2} \times AC \quad \dots[1/2] \\ \therefore BC &= \frac{1}{2} \times 14 \\ \therefore \mathbf{BC} &= \mathbf{7} \quad \dots[1/2] \quad [1] \end{aligned}$$

Q.2. (A) Complete the following activities and rewrite them.
(Any two) **[4]**

- (1) In the given figure, $\angle PQR$ is inscribed in the semicircle PQR . Then complete the following activity to find measure of $\angle PQR$.



Activity:

$$m(\text{arc PQR}) = 180^\circ \quad \dots \text{ (measure of semicircle)}$$

$$\therefore m(\text{arc PXR}) = \boxed{}$$

$$\angle PQR = \frac{1}{2} m(\text{arc } \boxed{}) \dots \dots \dots \boxed{}$$

$$= \frac{1}{2} \times 180^\circ$$

$$\therefore \angle PQR = \boxed{}$$

Solution:

$$m(\text{arc PQR}) = 180^\circ \quad \dots \text{ (measure of semicircle)}$$

$$\therefore m(\text{arc PXR}) = \boxed{180^\circ}$$

...[1/2]

$$\angle PQR = \frac{1}{2} m(\text{arc } \boxed{\text{PXR}}) \dots \boxed{\text{Inscribed angle theorem}}$$

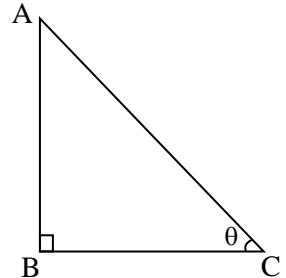
...[1/2] + [1/2]

$$= \frac{1}{2} \times 180^\circ$$

$$\therefore \angle PQR = \boxed{90^\circ}$$

...[1/2] [2]

(2) In $\triangle ABC$, $\angle B = 90^\circ$, $\angle C = \theta^\circ$, then complete the activity to derive the trigonometric identity.



Activity:

$$\text{In } \triangle ABC, \angle B = 90^\circ, \angle C = \theta^\circ$$

$$\therefore AB^2 + BC^2 = \boxed{}$$

.....(Pythagoras theorem)

$$\therefore \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2} \quad \dots \text{ (dividing by } AB^2 \text{)}$$

$$\therefore 1 + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$\text{But } \frac{\boxed{}}{AB^2} = \cot^2\theta \text{ and } \frac{AC^2}{\boxed{}} = \text{cosec}^2\theta$$

$$\therefore 1 + \boxed{} = \text{cosec}^2\theta$$

Solution:

In $\triangle ABC$, $\angle B = 90^\circ$, $\angle C = \theta^\circ$

$$\therefore AB^2 + BC^2 = \boxed{AC^2} \quad \dots \text{ (Pythagoras theorem) } \dots [1/2]$$

$$\therefore \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2} \quad \dots \text{ (dividing by } AB^2)$$

$$\therefore 1 + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$\text{But } \frac{\boxed{BC^2}}{AB^2} = \cot^2 \theta \text{ and } \frac{AC^2}{\boxed{AB^2}} = \operatorname{cosec}^2 \theta \quad \dots [1/2] + [1/2]$$

$$\therefore 1 + \boxed{\cot^2 \theta} = \operatorname{cosec}^2 \theta \quad \dots [1/2] \quad [2]$$

- (3) In $\triangle PQR$, if $PN = 12$, $NR = 8$, $PM = 15$, $MQ = 12$, then complete the following activity to justify whether seg NM is parallel to side RQ or not.

Activity:

In $\triangle PQR$,

$$\frac{PN}{NR} = \frac{12}{\boxed{}} = \frac{3}{2} \quad \dots \dots \text{ (I)}$$

$$\text{and } \frac{PM}{MQ} = \frac{15}{12} = \frac{\boxed{}}{4} \quad \dots \dots \text{ (II)}$$

$$\therefore \frac{PN}{NR} \neq \frac{PM}{MQ} \quad \dots \dots \text{ [from (I) and (II)]}$$

\therefore By $\boxed{}$

seg NM is $\boxed{}$ to side RQ .

Solution:

In $\triangle PQR$,

$$\frac{PN}{NR} = \frac{12}{\boxed{8}} = \frac{3}{2} \quad \dots \dots \text{ (I)} \quad \dots [1/2]$$

$$\text{and } \frac{PM}{MQ} = \frac{15}{12} = \frac{\boxed{5}}{4} \quad \dots \dots \text{ (II)} \quad \dots [1/2]$$

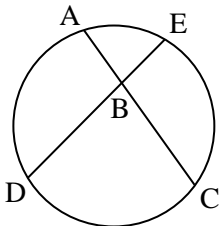
$$\therefore \frac{PN}{NR} \neq \frac{PM}{MQ} \quad \dots \dots \text{ [from (I) and (II)]}$$

∴ By **converse of basic proportionality theorem** ...[½]

seg NM is **not parallel** to side RQ. ...[½] [2]

Q.2. (B) Solve the following sub-questions. (Any four) [8]

(1) In the given figure, chord AC and chord DE intersect each other at point B.



If $\angle ABE = 108^\circ$ and $m(\text{arc AE}) = 95^\circ$, then find $m(\text{arc DC})$.

Solution:

$$\angle ABE = \frac{1}{2} [m(\text{arc AE}) + m(\text{arc DC})] \quad \dots[½]$$

$$\therefore 108^\circ = \frac{1}{2} [95^\circ + m(\text{arc DC})] \quad \dots[½]$$

$$\therefore 216^\circ = 95^\circ + m(\text{arc DC}) \quad \dots[½]$$

$$\therefore m(\text{arc DC}) = 216^\circ - 95^\circ = 121^\circ \quad \dots[½]$$

Ans. **$m(\text{arc DC}) = 121^\circ$** [2]

(2) Find the distance between the points P(-1, 1) and Q(5, -7).

Solution:

$$\text{Let } P(-1, 1) \equiv (x_1, y_1)$$

$$Q(5, -7) \equiv (x_2, y_2)$$

$$\therefore d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \dots(\text{Distance formula})\dots[½]$$

$$= \sqrt{[5 - (-1)]^2 + (-7 - 1)^2} \quad \dots[½]$$

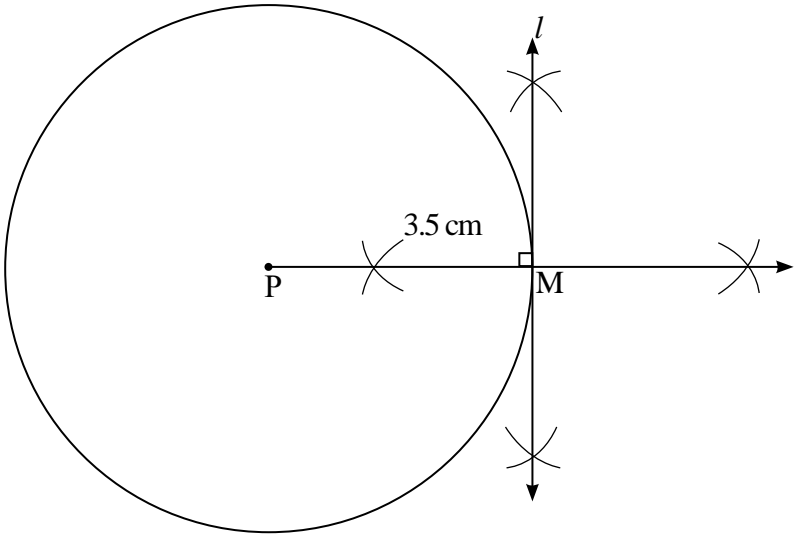
$$= \sqrt{(6)^2 + (-8)^2} \quad \dots[½]$$

$$= \sqrt{100}$$

$$\therefore \mathbf{d(P, Q) = 10} \quad \dots[½] [2]$$

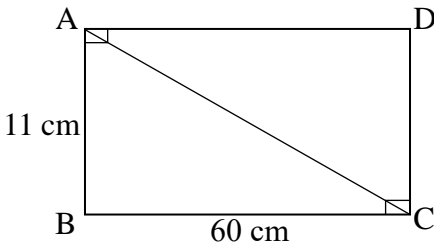
- (3) Construct a tangent to a circle with centre P and radius 3.5 cm at any point M on it.

Construction:



- To draw a circle with given radius and point M on it [1]
 - To draw a tangent at point M [1] [2]
- (4) Find the length of the diagonal of a rectangle having sides 11 cm and 60 cm.

Solution:



The sides of the rectangle intersect each other at 90° .

\therefore In $\triangle ABC$, $\angle ABC = 90^\circ$

\therefore By Pythagorus theorem,

$$AC^2 = AB^2 + BC^2 \quad \dots[1/2]$$

$$= (11)^2 + (60)^2 \quad \dots[1/2]$$

$$= 121 + 3600$$

$$= 3721 = (61)^2 \quad \dots[1/2]$$

$$\therefore \boxed{AC = 61}$$

Ans. Diagonal's length is 61 cm. ...[1/2] [2]

(5) If $\sin \theta = \frac{7}{25}$, then find the value of $\cos \theta$ and $\tan \theta$.

Solution:

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \dots[1/2]$$

$$\therefore \left(\frac{7}{25}\right)^2 + \cos^2 \theta = 1$$

$$\therefore \cos^2 \theta = 1 - \frac{49}{625}$$

$$\therefore \cos^2 \theta = \frac{576}{625}$$

$$\therefore \boxed{\cos \theta = \frac{24}{25}} \quad \dots[1/2]$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \dots[1/2]$$

$$= \frac{7}{25} \times \frac{25}{24}$$

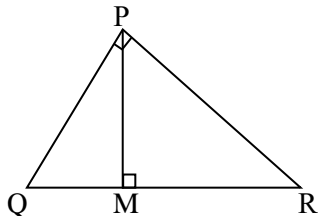
$$\therefore \boxed{\tan \theta = \frac{7}{24}} \quad \dots[1/2] [2]$$

Q.3. (A) Complete the following activities and rewrite them.

(Any one)

[3]

- (1) In the given figure, $\angle QPR = 90^\circ$,
 seg $PM \perp$ seg QR and Q - M - R .
 $PM = 10$, $QM = 8$, then complete
 the following activity to find the
 value of QR .



Activity:

In ΔPQR , $\angle QPR = 90^\circ$ and seg $PM \perp$ seg QR .

$$\therefore PM^2 = \boxed{} \times MR \dots\dots\dots \boxed{}$$

$$\therefore (\boxed{})^2 = 8 \times MR$$

$$\therefore \frac{100}{8} = MR$$

$$\therefore \square = MR$$

$$\text{Now } QR = QM + MR \dots\dots\dots(\because Q-M-R)$$

$$\therefore QR = 8 + \square$$

$$\therefore QR = \square$$

Solution:

In ΔPQR , $\angle QPR = 90^\circ$ and seg $PM \perp$ seg QR .

$$\therefore PM^2 = \square \times MR \dots \text{Geometric mean theorem} \dots\dots\dots[1/2] + [1/2]$$

$$\therefore (10)^2 = 8 \times MR \dots\dots\dots[1/2]$$

$$\therefore \frac{100}{8} = MR$$

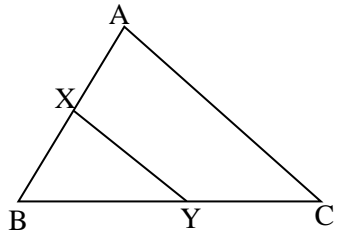
$$\therefore \square = MR \dots\dots\dots[1/2]$$

$$\text{Now } QR = QM + MR \quad (\because Q-M-R)$$

$$\therefore QR = 8 + \square \dots\dots\dots[1/2]$$

$$\therefore QR = \square \dots\dots\dots[1/2] \quad [3]$$

(2) In the given figure, in ΔABC , seg $XY \parallel$ side AC , $A-X-B$, $B-Y-C$. If $2AX = 3BX$ and $XY = 9$ then complete the following activity to find the value of AC .



Activity:

$$2AX = 3BX \quad \dots\dots\dots(\text{given})$$

$$\therefore \frac{AX}{BX} = \frac{\square}{\square}$$

$$\therefore \frac{AX + BX}{BX} = \frac{3 + 2}{2} \quad \dots\dots\dots\text{componendo}$$

$$\therefore \frac{AB}{BX} = \frac{5}{2} \quad \dots\dots\dots(\text{I})$$

$\Delta BCA \sim \Delta BYX$ \square test of similarity

$$\therefore \frac{BA}{BX} = \frac{AC}{\square} \quad \dots\dots\dots\text{c.s.s.t.}$$

$$\therefore \frac{5}{2} = \frac{AC}{\boxed{}} \dots\dots\dots\text{from (I)}$$

$$\therefore AC = \boxed{}$$

Solution:

$$2AX = 3BX \dots\dots\dots\text{given}$$

$$\therefore \frac{AX}{BX} = \frac{\boxed{3}}{\boxed{2}}$$

$$\therefore \frac{AX + BX}{BX} = \frac{3 + 2}{2} \dots\dots\dots\text{componendo ...}[\frac{1}{2}] + [\frac{1}{2}]$$

$$\therefore \frac{AB}{BX} = \frac{5}{2} \dots\dots\dots\text{(I)}$$

$$\triangle BCA \sim \triangle BYX \dots\dots \boxed{AA} \text{ test of similarity ...}[\frac{1}{2}]$$

$$\therefore \frac{BA}{BX} = \frac{AC}{\boxed{XY}} \dots\dots\dots\text{c.s.s.t} \dots[\frac{1}{2}]$$

$$\therefore \frac{5}{2} = \frac{AC}{\boxed{9}} \dots\dots\dots\text{from (I)} \dots[\frac{1}{2}]$$

$$\therefore AC = \boxed{22.5} \dots[\frac{1}{2}] [3]$$

Q.3. (B) Solve the following sub-questions. (Any two) [6]

(1) Prove that:

$$\sec \theta + \tan \theta = \frac{\cos \theta}{1 - \sin \theta}$$

Solution:

$$\begin{aligned} \text{RHS} &= \frac{\cos \theta}{1 - \sin \theta} \\ &= \frac{\cos \theta}{1 - \sin \theta} \times \frac{1 + \sin \theta}{1 + \sin \theta} \dots[\frac{1}{2}] \end{aligned}$$

$$= \frac{\cos \theta (1 + \sin \theta)}{1 - \sin^2 \theta} \dots[\frac{1}{2}]$$

$$= \frac{\cos \theta (1 + \sin \theta)}{\cos^2 \theta} \dots\dots\dots (\because \sin^2 \theta + \cos^2 \theta = 1) \dots[\frac{1}{2}]$$

$$= \frac{\cos \theta}{\cos \theta} \times \frac{1 + \sin \theta}{\cos \theta}$$

$$= \frac{1 + \sin \theta}{\cos \theta} \quad \dots[1/2]$$

$$= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \quad \dots[1/2]$$

$$= \sec \theta + \tan \theta \quad \dots[1/2]$$

\therefore **RHS = LHS**

Hence proved

(2) Find the co-ordinates of the centroid of the triangle whose vertices are (4, 7), (8, 4), (7, 11).

Solution:

Let (4, 7) \equiv (x_1, y_1)

(8, 4) \equiv (x_2, y_2)

(7, 11) \equiv (x_3, y_3)

Let (x, y) be the co-ordinates of the centroid.

$$\therefore (x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \quad [1]$$

$$= \left(\frac{4 + 8 + 7}{3}, \frac{7 + 4 + 11}{3} \right) \quad [1]$$

$$\therefore (x, y) = \left(\frac{19}{3}, \frac{22}{3} \right) \quad \dots[1/2]$$

Ans. The co-ordinates of the centroid are $\left(\frac{19}{3}, \frac{22}{3} \right)$. $\dots[1/2]$ [3]

(3) Prove that ‘opposite angles of a cyclic quadrilateral are supplementary’.

Solution:

Given: \square ABCD is a cyclic quadrilateral. $\dots[1/2]$

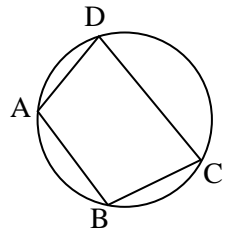
To prove: $\angle B + \angle D = 180^\circ$

$\angle A + \angle C = 180^\circ$

Proof:

By the inscribed angle theorem,

$$m\angle ADC = \frac{1}{2} m(\text{arc ABC}) \dots\dots\dots (1) \quad \dots[1/2]$$



and $m\angle ABC = \frac{1}{2} m(\text{arc ADC}) \dots\dots\dots (2)$

\therefore Adding equations (1) and (2),

$$m\angle ADC + m\angle ABC = \frac{1}{2} m(\text{arc ABC}) + \frac{1}{2} m(\text{arc ADC}) \dots[1/2]$$

$$= \frac{1}{2} [m(\text{arc ABC}) + m(\text{arc ADC})]$$

$$\therefore m\angle D + m\angle B = \frac{1}{2} \times 360^\circ \text{ (Measure of a complete circle)} \dots[1/2]$$

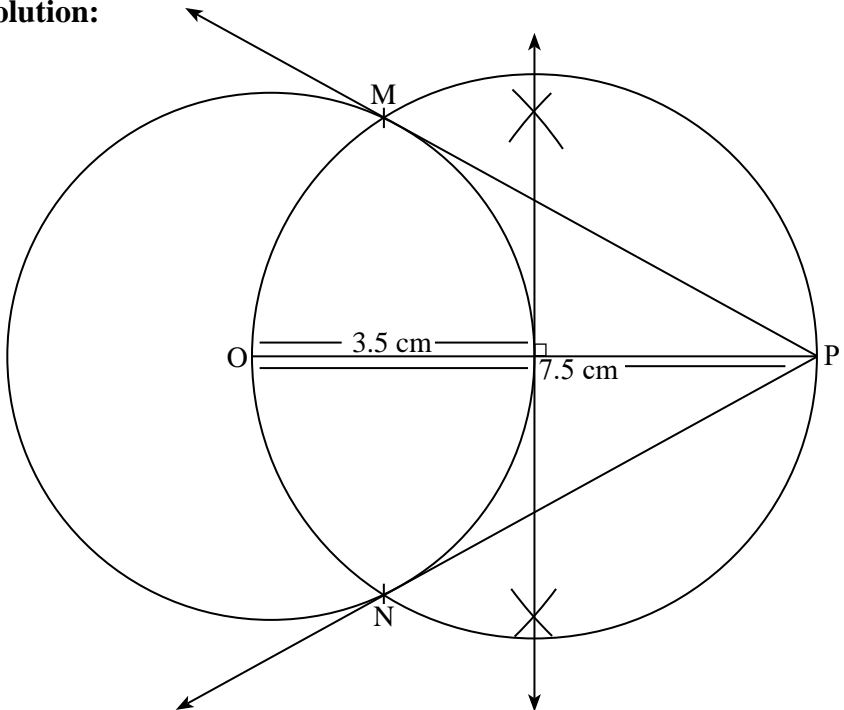
$$\therefore \angle B + \angle D = 180^\circ \dots[1/2]$$

Similarly, $\angle A + \angle C = 180^\circ \dots[1/2] [3]$

Ans. Hence proved that opposite angles of a cyclic quadrilateral are supplementary.

(4) Draw a circle with centre 'O' and radius 3.5 cm. Take a point P at a distance of 7.5 cm from the centre. Draw tangents to the circle from point P.

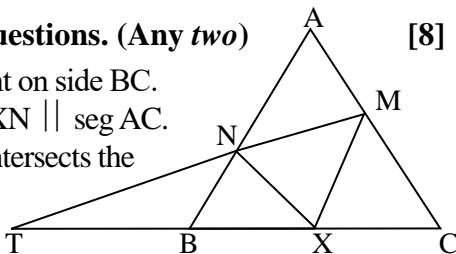
Solution:



- To draw a circle with centre O and radius 3.5 cm ...[½]
- To take point P at a distance of 7.5 cm from O ...[½]
- To draw the perpendicular bisector of seg OP ...[½]
- To locate points M and N ...[½]
- To draw the tangent segments PM and PN [1] [3]

Q.4. Solve the following sub-questions. (Any two) [8]

- (1) In $\triangle ABC$, point X is any point on side BC.
 Seg XM \parallel seg AB and seg XN \parallel seg AC.
 Extend seg MN such that it intersects the
 extended side BC in point T.



Then prove that:

$$TX^2 = TB \times TC$$

Proof:

In $\triangle TMC$, seg NX \parallel seg MC(given)

$$\therefore \frac{TN}{NM} = \frac{TX}{XC} \dots\dots\dots(1) \quad (\text{Basic proportionality theorem})$$

...[½]

$\triangle TMX$, seg XM \parallel seg NB(given)

$$\therefore \frac{TN}{NM} = \frac{TB}{BX} \dots\dots\dots(2) \quad (\text{Basic proportionality theorem})$$

...[½]

\therefore From (1) and (2),

$$\frac{TX}{XC} = \frac{TB}{BX} \dots\dots\dots$$

...[½]

$$\therefore \frac{XC}{TX} = \frac{BX}{TB} \dots\dots\dots(\text{By invertendo}) \dots\dots\dots$$

...[½]

$$\therefore \frac{XC + TX}{TX} = \frac{BX + TB}{TB} \dots\dots\dots(\text{By componendo}) \dots\dots\dots$$

...[½]

$$\therefore \frac{TC}{TX} = \frac{TB}{TX} \dots\dots\dots(\because T-X-C \text{ and } T-B-X) \dots\dots\dots$$

...[½] + [½]

$$\therefore \boxed{TX^2 = TB \times TC} \dots\dots\dots$$

...[½] [4]

Hence proved

- (2) Draw a triangle ABC, right angle at B such that AB = 3 cm, BC = 4 cm. Now construct ΔPBQ similar to ΔABC , each of whose sides are $\frac{7}{4}$ times the corresponding sides of ΔABC .

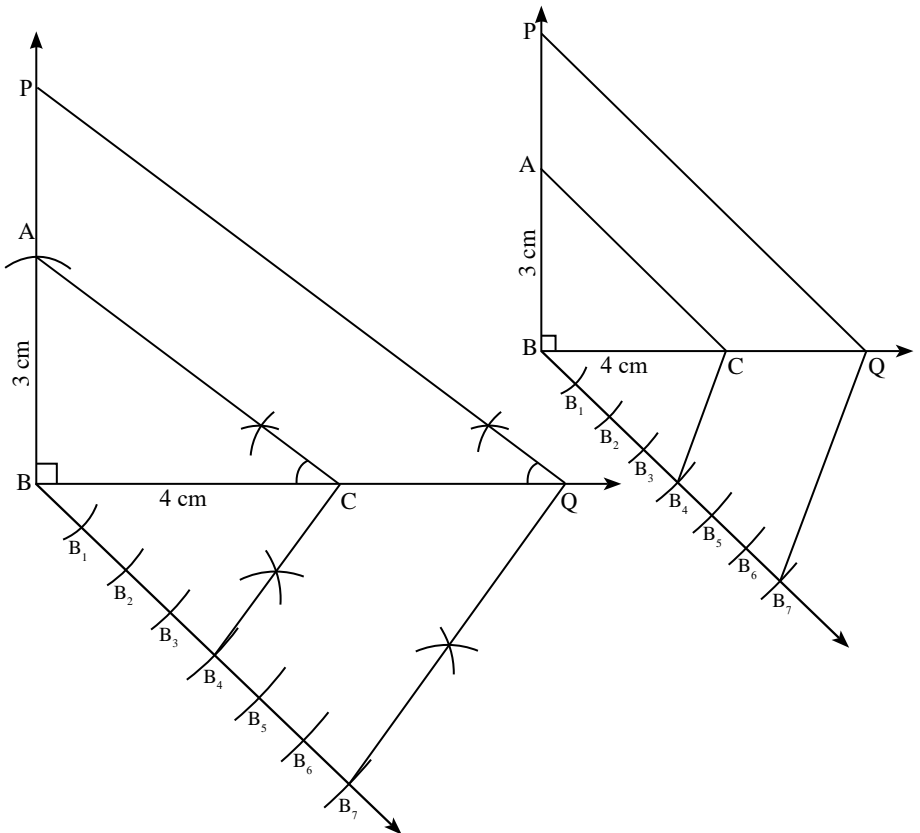
Solution:

Given: In ΔABC ,

$$AB = 3 \text{ cm}, \angle B = 90^\circ, BC = 4 \text{ cm}$$

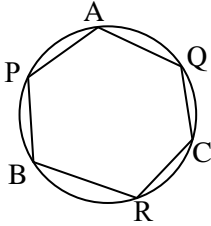
$$\Delta ABC \sim \Delta PBQ$$

$$\therefore PB = \frac{7}{4} AB, BQ = \frac{7}{4} BC, PQ = \frac{7}{4} AC$$



- Analytical figure [1]
- Construction of ΔABC [1]
- Construct $B_4C \parallel B_7Q$ and $AC \parallel PQ$ [1]
- Construct ΔPBQ [1] [4]

(3)



In the given figure, points A, P, B, R, C, Q are on the circle. After joining the given points as shown in the figure, they form a hexagon. Then prove that:

$$\angle APB + \angle BRC = 360^\circ - \angle AQC$$

Proof:

By inscribed angle theorem,

$$\begin{aligned} \angle APB &= \frac{1}{2} \times m(\text{arc ACB}) \\ &= \frac{1}{2} [m(\text{arc AQ}) + m(\text{arc QC}) + m(\text{arc CR}) + m(\text{arc RB})] \\ &\quad \dots[\frac{1}{2}] + [\frac{1}{2}] \\ &\quad \dots\dots\dots (1) \end{aligned}$$

and

$$\begin{aligned} \angle BRC &= \frac{1}{2} \times m(\text{arc BAC}) \\ &= \frac{1}{2} [m(\text{arc BP}) + m(\text{arc PA}) + m(\text{arc AQ}) + m(\text{arc QC})] \\ &\quad \dots[\frac{1}{2}] \\ &\quad \dots\dots\dots (2) \end{aligned}$$

\therefore Adding (1) and (2),

$$\begin{aligned} &\angle APB + \angle BRC \\ &= \frac{1}{2} m(\text{arc AQ}) + \frac{1}{2} m(\text{arc QC}) + \frac{1}{2} m(\text{arc CR}) + \frac{1}{2} m(\text{arc RB}) \\ &\quad + \frac{1}{2} m(\text{arc BP}) + \frac{1}{2} m(\text{arc PA}) + \frac{1}{2} m(\text{arc AQ}) + \frac{1}{2} m(\text{arc QC}) \\ &\quad \dots[\frac{1}{2}] \\ &= m(\text{arc AQ}) + m(\text{arc QC}) + \frac{1}{2} [m(\text{arc CR}) + m(\text{arc RB}) \\ &\quad + m(\text{arc BP}) + m(\text{arc AP})] \\ &= m(\text{arc AQ}) + m(\text{arc QC}) + \frac{1}{2} m(\text{arc ABC}) \\ &\quad \dots\dots\dots (3) \quad \dots[\frac{1}{2}] \end{aligned}$$

Now, $m(\text{arc AQ}) + m(\text{arc QC}) + m(\text{arc ABC}) = 360^\circ$
 (Measure of a complete circle)

$$\begin{aligned} \therefore m(\text{arc AQ}) + m(\text{arc QC}) &= 360^\circ - m(\text{arc ABC}) \quad \dots[\frac{1}{2}] \\ &\quad \dots\dots\dots (4) \end{aligned}$$

∴ From (3) and (4),

$$\begin{aligned}\angle APB + \angle BRC &= 360^\circ - m(\text{arc } ABC) + \frac{1}{2} m(\text{arc } ABC) \\ &= 360^\circ - \frac{1}{2} m(\text{arc } ABC)\end{aligned}$$

$$\text{But } \angle AQC = \frac{1}{2} m(\text{arc } ABC) \text{ (Inscribed angle theorem)}$$

..... (5) ...[½]

∴ Using (5),

$\angle APB + \angle BRC = 360^\circ - \angle AQC$...[½] [4]
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Hence proved

Q.5. Solve the following sub-questions. (Any one) [3]

(1) $\triangle ABC$ and $\triangle PQR$ are equilateral triangles with altitudes $2\sqrt{3}$ and $4\sqrt{3}$ respectively, then

(a) Find the length of side AB and side PQ. [1]

(b) Find $\frac{A(\triangle ABC)}{A(\triangle PQR)}$. [1]

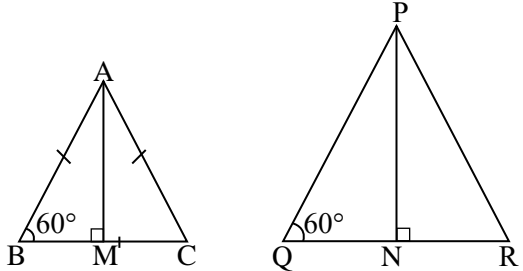
(c) Find the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle PQR$. [1]

Solution:

Let AM and PN be the altitudes of $\triangle ABC$ and $\triangle PQR$ respectively.

∴ $AM = 2\sqrt{3}$ and

∴ $PN = 4\sqrt{3}$



(a) In $\triangle ABM$,

$\angle B = 60^\circ$ (∵ $\triangle ABC$ is an equilateral \triangle)

$AM \perp BC$

∴ $\angle M = 90^\circ$

∴ $\angle A = 30^\circ$ (Remaining angle of $\triangle ABM$)

∴ $\triangle ABM$ is a $30^\circ - 60^\circ - 90^\circ$ triangle.

∴ By $30^\circ - 60^\circ - 90^\circ$ triangle theorem,

$$\text{side opposite the } 60^\circ \text{ angle} = \frac{\sqrt{3}}{2} \times \text{hypotenuse}$$

$$\therefore AM = \frac{\sqrt{3}}{2} \times AB$$

$$\therefore AB = \frac{2}{\sqrt{3}} \times 2\sqrt{3}$$

$$\therefore \boxed{AB = 4} \quad \dots[1/2]$$

Similarly, ΔPQN is also a $30^\circ - 60^\circ - 90^\circ$ triangle.

∴ By $30^\circ - 60^\circ - 90^\circ$ triangle theorem,

$$PN = \frac{\sqrt{3}}{2} \times PQ \quad \dots \text{(Side opposite the } 60^\circ \text{ angle)}$$

$$\therefore 4\sqrt{3} = \frac{\sqrt{3}}{2} \times PQ$$

$$\therefore \boxed{PQ = 8} \quad \dots[1/2] \quad [1]$$

(b) Equilateral triangles are similar.

$$\therefore \Delta ABC \sim \Delta PQR$$

∴ By the theorem on areas of similar triangles,

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 \quad \dots[1/2]$$

$$= \left(\frac{4}{8}\right)^2$$

$$= \left(\frac{1}{2}\right)^2$$

$$\therefore \boxed{\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{1}{4}} \quad \dots[1/2] \quad [1]$$

$$(c) \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB + BC + AC}{PQ + QR + PR}$$

But ΔABC and ΔPQR are equilateral Δ .

$$\begin{aligned} \therefore \frac{P(\Delta ABC)}{P(\Delta PQR)} &= \frac{3AB}{3PQ} && \dots[1/2] \\ &= \frac{4}{8} = \frac{1}{2} \end{aligned}$$

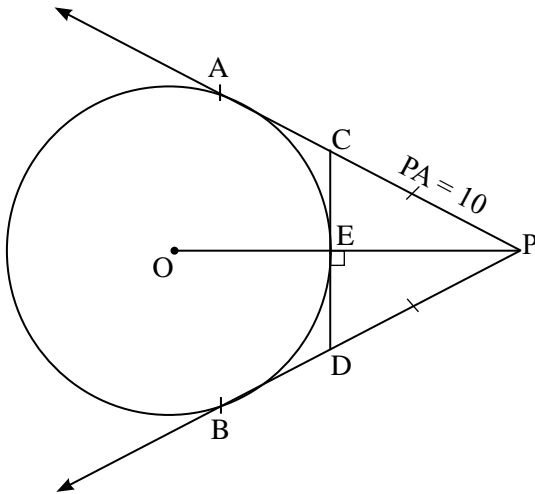
$$\therefore \frac{P(\Delta ABC)}{P(\Delta PQR)} = \frac{1}{2} \quad \dots[1/2] \quad [3]$$

(2) In a circle with centre O, PA and PB are tangents from an external point P. E is the point on the circle such that O-E-P. The tangent drawn at E intersects PA and PB in points C and D respectively. If PA = 10, then write answers to the following questions:

- (a) Draw the suitable figure using given information. [1/2]
- (b) Write the relation between seg PA and seg PB. [1/2]
- (c) Find the perimeter of ΔPCD . [2]

Solution:

(a)



...[1/2]

(b) PA = PB (\because The tangents drawn from an exterior point to a circle are equal.) ...[1/2]

(c) \therefore Perimeter of $\triangle PCD = PC + CD + DP \dots\dots\dots(1)\dots[\frac{1}{2}]$

But $CD = CE + ED \dots\dots\dots(\because C - E - D)$

Also, $CE = CA$ and $ED = BD$
 $\dots\dots$ (Tangents drawn from an exterior point to the circle) $\dots[\frac{1}{2}]$

$\therefore CD = CA + BD \dots\dots\dots(2)$

\therefore From (1) and (2),

$\therefore P(\triangle PCD) = PC + CA + BD + DP$
 $= PA + PB \dots\dots\dots (\because P - C - A \text{ and } P - D - B)$
 $= PA + PA \dots\dots\dots [\text{using (b)}]$
 $= 10 + 10 (\because PA = 10) \dots[\frac{1}{2}]$

$\therefore P(\triangle PCD) = 20$

Ans. Perimeter of $\triangle PCD$ is 20. $\dots[\frac{1}{2}] [3]$

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